An Approach for Optimization of Gas-Lift Allocation to a Group of Wells and Oilfield*

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Absract: The paper summarizes the current status of the optimization of gas allocation to a group of wells, and analyzes the characteristics of all types of the gas-lift performance curves. Considering the practicability to fields and the rationality, the paper fits the scattered points ahead of maximum value in the gas lift performance curve as quadratic equation. The well group performance curve is defined as the relationship between the gas injection rate and the maximum oil production rate. The paper consummates the well group optimization model of gas allocation and develops the oilfield (with n well groups) optimization model of gas allocation, and gives a new method as well penalty function(SUMT) to solve the non-linear optimization model of gas allocation. To improve the rate of solving the model, a new approach named maximum gas allocation is proposed to guess the initial gas injection rates. The penalty function is capable of accommodating both the equality constraints and inequality constraints. As a rule, the convergence rate is affected only by an initial guess for the gas injection rates. By using the new method which the paper puts forward, well groups and oilfield gas allocation model is solved and the optimal gas allocatin results of well group A and B were provided in the end.

Keywords:*optimization gas allocation, performance curve, mathematical model, non-linear constraint , discipline function*

INTRODUCTION

In the theory and method of gas allocation optimization, Mayhill[1] was the pioneer who analyzed the relationship between the gas injection rate and the production oil rate, and called the relationship “gas lift performance curve(GLPC)”. He defined the optimal gas injection rate as the rate at which an incremental cost for gas injection is just equal to some percentage of the incremental revenue. Radden[2] et al presented an analytical approach for determining the optimal gas injection rate in continuous gas lift wells system, They also developed a program to peform calculations for the gas allocation. Gomez[3] proposed that the gas lift performance curve was fitted as the second degree polynomial. The polynomial was used to determine which well would increase the largest amount of oil under the incremental same gas injection rate, and this well would then be allocated the incremental amount of gas. Kanu[4] et al used firstly the equal slope allocation method and presented the formulation of the economic slope. N.Nishikiori[5] et al adopted a quasi-Newton non-linear optimization technique to find the optimal gas injection point, but the constraint considered was only lower limit. Hong[6] researched the effect of several correlative variable on the continuous gas lift system. He employed a cubic spline interpolation technique for the estimation of the gas lift performance curve. The search algorithm proposed by Buitrago[7] et al is a combination of stochastic domain exploration and a heuristic calculation of a descent direction, which can avoid stopping the algorithm at a local optimum.

In summary, in the continuous gas lift system optimization of gas allocation is categorized into

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two different approaches:

1. The first one is based on economical objective. The optimization gas injection rate is the point at which the cost of an incremental amount of injection gas is just equal to the profit from an incremental increase of oil production. The methods proposed by Mayhill,Radden et al and Kanu et al are based on this.
2. The second one is based on obtaining the maximum oil production from a group of wells under the same gas injection rate.the optimal gas injection rate is then determined.the procedures developed by Hong,Gomez,N.Nishikiori et al and Buitrago et al follow this approach.

The method to determine optimal gas allocation proposed by Hong is similar to the one Gomez developed, nobbut the procedures to generate the gas-lift performance curve are different. the method is virtually a kind of enumerating technique, which provides unsatisfying the speed and the accuracy. N.Nishikiori et al used a quasi-Newton technique, which is very complicated on the constraints, and the speed of solving the model is not perfect yet. The disadvantage of method proposed by Buitrago[7] et al is similar to that of N.Nishikiori. Aiming at the maximum oil production,the paper firstly allocates the gas injection rates for the single well, and plot the gas lift well group performance curve, then for the oilfield with the exterior point law of penalty function.

MATHEMATICAL MODEL

1.Gas-lift well performance curve

The shape of the gas lift performance curve depends on the oil production response to gas injection. Curve A in Fig.1 belongs to a well capable of producing naturally. Curve B represents the behavior of a well that have the strong capability of producing but not naturally, it can produce as long as gas is injected into.Curve C belongs to a well requiring a considerable amount of gas injection to start production. Curve D is similar to Curve C, except a jump.

The gas lift performance curve can be obtained by the nodal analysis or by measuring the rate of gas injection and the rate of oil production in oilfield. The point obtained by approaches above are scattered. It is necessary to fit these scattered points as a curve equation in order to allocate gas injection rate for each well with optimization technique that computer is easy to come true. In the research gas lift performance curve is matched as the second degree multinomial by the least square method. Gas lift performance curve is expressed as follows:

$$q_o = Aq_g^2 + Bq_g + C \qquad (1)$$

In view of the restricted gas injection rate,the rate allocated for the single well is often less than the maximum value. As a result, only those points ahead of maximum value in the gas lift performance curve were matched, by which the precision will be enhanced clearly.

2. The well group gas-lift performance curve

The well group performance curve is defined as the relationship between the gas injection rate and the maximum oil production.Comparatively speaking with that of the single well , as a rule there is not the shape of curve C or curve D. For special case,the method is similar to that of the single well. And the well group performance curve presents only a climbing trend.

The well group gas lift performance curve which are scattered points can be attained by gas allocation optimization for the well. After performing a large number of calculations and analysises,we discover that the shape of curve is consistent to the second degree polynomial curve.

3.Optimization model of gas allocation to a group of wells

A group of wells with n wells, forms an aggregate N. The total oil production Q_{oTOT}, which is the sum of the individual oil production rates q_{oi} can be considered as a function of the individual gas injection rates q_{gi}.Mathematical formulation is expressed as follows:

$$Q_{oTOT} = \sum_{i=1}^{n} q_{oi} = f\left(\mathrm{q}_{\mathrm{g}}\right) = f\left(q_{g1}, q_{g2}, ..., q_{gn}\right) \quad (2)$$

where $\mathbf{q_g}$ is represented by an n-dimensional column vector:

$$\mathrm{q}_{\mathrm{g}} = \left(q_{g1}, q_{g2}, ..., q_{gn}\right)^T \quad (3)$$

The maximum oil production of wells is expressed as:

$$MaxQ_{oTOT} = Maxf\left(q_{g1}, q_{g2}, ..., q_{gn}\right) \quad (4a)$$

substituting Eq.(1) into Eq.(4a),we have

$$MaxQ_{oTOT} = Max\sum_{i=1}^{n}\left(A_i q_{gi}^2 + B_i q_{gi} + C_i\right) \quad (4b)$$

in the Eq.(1), the gas injection rate corresponded to the individual well maximum oil production is expressed as:

$$q_{gi\max} = -\frac{B_i}{2A_i} \quad (5)$$

So the total gas injection rate corresponded to the total maximum oil production is expressed as:

$$Q_{g\max} = \sum_{i=1}^{n} -\frac{B_i}{2A_i} \quad (6)$$

So we draw a conclusion from Eq.(6) as follows:

The total gas injection rate don't generate the constraint to the system if the maximum injection gas volume available Q_{gAT} is not less than the rate Q_{gmax},and vice versa. The total gas injection rate subject to the following constraint:

$$\sum_{i=1}^{n} q_{gi} = Q_{gAT} \quad (7a)$$

where $Q_{gAT} \le Q_{g\max}$.

Curve A and curve B in Fig.1 form an aggregate I,the constraint to the individual well is expressed as follows:

$$0 \le q_{gi} \le q_{gi\max} \quad (7b)$$

Curve C and curve D in Fig.1 form an aggregate J,the constraint is expressed as follows:

$$q_{gi\min} \le q_{gi} \le q_{gi\max}$$

or

$$q_{gi} = 0 \qquad (7c)$$

where $q_{gimin}>0$.

In summary,we have the following optimization gas allocation model based on the maximum oil production.

$$\begin{cases} MaxQ_{oTOT} = Max\sum_{i=1}^{n}\left(A_i q_{gi}^2 + B_i q_{gi} + C_i\right) \\ \sum_{i=1}^{n} q_{gi} = Q_{gAT} \\ 0 \le q_{gk} \le q_{gk\max} \\ q_{gj\min} \le q_{gj} \le q_{gj\max} \quad \text{or} \quad q_{gj} = 0 \end{cases} \qquad (8)$$

where $k \in \mathrm{I}$， $j \in \mathrm{J}$，$\mathrm{J} \cup \mathrm{I} = \mathrm{N}$ ， $\mathrm{J} \cap \mathrm{I} = \Phi$.

4.Optimization model of gas allocation to oilfield

There are m well groups,the total oil production rates is Q_{oT} from well groups with the total gas injection rates Q_{gj}, in a similar way to developing a group of wells optimization model,we have:

$$\begin{cases} MaxQ_{oT} = Max\sum_{j=1}^{m}\left(A_j Q_{gj}^2 + B_j q_{gj} + C_j\right) \\ \sum_{j=1}^{m} Q_{gj} = Q_{gT} \\ 0 \le Q_{gj} \le Q_{gj\max} \qquad j = 1, 2, \ldots, m \end{cases} \qquad (9)$$

NON-LINEAR OPTIMIZATION TECHNIQUE

The numerical calculation techniques which solves the non-linear programming problem with the constraints usually include the direct method and indirect one. such as the sawtooth method, the projection gradient method, the complex method as well as the genetic algorithm of modern times optimization technique, belong to the former, whereas the penalty function and the augmentation multiplier belong to the later. The penalty function is usually adopted to solve the optimization problem with the constraints.which is transformed nonrestraint by appending the penalty factors on the adaptive value function. It is easy to do.In this research the exterior point technique of the penalty function is adopted to solve Eq.(8) and Eq(9).

1 Disassembling Constraint

Firstly,given that the gas injection rate q_{gj} is not equal to zero,the constraints are newly expressed as follows:

$$\begin{cases} \sum_{i=1}^{n} q_{gi} = Q_{gAT} \\ q_{gi\max} - q_{gi} \geq 0 \\ q_{gi} - q_{gi\min} \geq 0 \end{cases} \qquad (10)$$

where $q_{gimax}>0$， $q_{gimin}\geq 0$.

If the gas injection rate q_{gj} is equal to zero, in other words, the gas allocation rate is zero to the curve C and the curve D in Fig.1, then, the constraints are newly expressed as follows:

$$\begin{cases} \sum_{i=1}^{k} q_{gi} = Q_{gAT} \\ 0 \leq q_{gi} \leq q_{gi\max} \qquad i = 1, 2, ..., k \end{cases} \qquad (11)$$

2 Basic Procedure

1) Setting the initial estimate $\mathbf{q^0_g}$,the initial penalty factor δ ,the magnifing sequence $\mathbf{c^k}$,the allowable tolerance ε ,and k=1;
2) Setting the initial estimate $\mathbf{q_g}^{(k-1)}$,solving the following non-constraints problem.

   $$\max f\left(\mathrm{q_g}\right) + F_1\left(\mathrm{q_g}, \delta\right) + F_2\left(\mathrm{q_g}, \delta\right)$$

   Given the maximum $\mathbf{q_g}^{k}$, where F_1,F_2 is respectively the penalty factors of equation and inequation constraints;
3) If $F_1\left(\mathrm{q_g^k}, \delta\right) + F_2\left(\mathrm{q_g^k}, \delta\right) < \varepsilon$, terminate computation,the approximate solution $\mathrm{q_g}^{k}$;or setting

   $\delta^k = c^k \delta$,back to 2).

3 Initial Estimate

The penalty function optimization technique requires an initial guess for the gas injection rates $q^0{}_g$,which must satisfy all the constraints defined by Eqs.(10) or Eqs.(11), The initial guesses have direct effect on the speed of convergence, even converging or not.there are the following procedures for the initial guesses.

Productivity Index Allocation:One way is to allocate gas in proportion to the productivity index for each well,i.e.

$$q_{gi}^0 = Q_{gAT} \frac{PI_i}{\sum_{i=1}^{n} PI_i} \qquad (12)$$

Maximum Liquid Production Allocation[4]:Another possible way is allocate gas in proportion to the maximum oil production rate possible for each well,i.e.

$$q_{gi}^0 = Q_{gAT} \frac{q_{oi\max}}{\sum_{i=1}^{n} q_{oi\max}} \qquad (13)$$

Maximum Gas Allocation:A new procedure established in this study is to allocate gas in proportion to the maximum injection rate for each well,i.e.

$$q_{gi}^{0} = Q_{gAT} \frac{q_{g_i \max}}{\sum_{i=1}^{n} q_{g_i \max}} \qquad (14)$$

We can obtain easily the maximum gas allocation rate of each well,the sum of which is the maximum gas allocation rate of wells. Then compute the initial guess according to Eq.(14).the initial guess is more close to the optimal value.

COMPUTATIONAL RESULTS

Two well groups A and B(shown in Fig.2) include ten wells and eight wells respectively(shown Fig.3 and Fig.4). The coefficients of the performance curve equations are listed into Tab.1 and Tab.2,the results are listed into Tab.3～Tab.4. The well8 in the well group B belongs to curve C or curve D in Fig.1, if the gas allocation rate of it is zero,the results in Tab.5. By comparing Tab.4 with Tab.5 we can draw a conclusion as following: If the total injection rates are low, and the gas allocation rates of the curve C or curve D wells are equal to zero, the total oil production rates of wells are likely to be more. While the total injection rates are more abundant but less than the total maximum gas injection rates, the conclusion is possible reverse. So we should consider them respectively when all curves in Fig1 exist in a group of wells by program to select the constraints .The well group A and B gas lift performance curves are shown respectively in Fig.5 and Fig.6. The results are listed in Tab.6 with the different gas injection rates.

Tab1 the coefficients of the well group A gas lift performance curve equations

Name	A	B	C
Well1	-5	50	0.1
Well2	-8	40	10
Well3	-4	30	2
Well4	-2	16	20
Well5	-4.1	16	40
Well6	-8	70	0.5
Well7	-7.4	46	0.7
Well8	-11	60	1
Well9	-10	100	0.3
Well10	-10	60	30

Tab2 the coefficients of the well group B gas lift performance curve equations

Name	A	B	C
Well1	-3	40	20
Well2	-2	18	12.5
Well3	-4	28.6	40
Well4	-2	18	15
Well5	-1.1	6.8	40
Well6	-5	40	18
Well7	-4.2	44	31
Well8	-8	50	-50

Tab3 the results of gas allocation for the well group A with the different gas injection rates

Name	total gas injection rate（$10^4 m^3/d$）、total oil production（m^3/d）							
	10		20		30		maximum	
	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}
Well1	0.00	0.10	4.71	124.67	4.42	123.43	5.00	125.10
Well2	0.00	10.00	0.94	40.59	2.14	58.95	2.50	60.00
Well3	0.71	21.26	1.88	44.33	3.03	56.16	3.75	58.25

Well4	0.00	20.00	0.00	20.00	2.55	47.81	4.00	52.00
Well5	0.00	40.00	0.00	40.00	1.25	53.57	1.95	55.61
Well6	2.35	120.96	2.94	137.20	4.01	152.58	4.38	153.63
Well7	0.00	0.70	1.08	41.65	2.72	71.06	3.11	72.19
Well8	1.57	68.07	2.14	79.02	2.46	82.06	2.73	82.82
Well9	3.48	227.30	3.95	239.35	4.71	249.46	5.00	250.30
Well10	1.88	107.54	2.35	115.82	2.71	119.16	3.00	120.00
total	9.99	615.93	19.99	882.633	29.99	1014.23	35.41	1029.89

Tab4 the results of gas allocation for the well group B with the different gas injection rates

Name	total gas injection rate（$10^4m^3/d$）、total oil production（m^3/d）							
	10		20		30		maximum	
	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}
Well1	3.31	119.51	4.86	143.59	6.15	152.56	6.67	153.33
Well2	0.00	12.50	1.80	38.38	3.73	51.82	4.50	53.00
Well3	0.00	40.00	2.22	83.81	3.19	90.53	3.58	91.12
Well4	0.00	15.00	1.80	40.88	3.73	54.32	4.50	55.50
Well5	0.00	40.00	0.00	40.00	1.69	48.36	3.09	50.51
Well6	1.99	77.71	2.92	92.15	3.69	97.53	4.00	98.00
Well7	2.84	122.08	3.95	139.28	4.87	145.68	5.24	146.24
Well8	1.87	15.44	2.45	24.47	2.93	27.83	3.13	28.13
total	10.01	442.24	20.00	602.57	29.99	668.63	34.61	675.83

Tab5 the results of gas allocation for the well group B with the different gas injection rates(Well8=0)

Name	total gas injection rate（$10^4m^3/d$）、total oil production（m^3/d）							
	10		20		30		maximum	
	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}
Well1	3.57	124.62	5.27	147.47	6.49	153.24	6.67	153.33
Well2	0.00	12.50	2.40	44.20	4.23	52.86	4.50	53.00
Well3	1.26	69.58	2.53	86.72	3.44	91.05	3.58	91.12
Well4	0.00	15.00	2.40	46.70	4.23	55.36	4.50	55.50
Well5	0.00	40.00	0.00	40.00	2.60	50.25	3.09	50.51
Well6	2.14	80.77	3.16	94.48	3.89	97.94	4.00	98.00
Well7	3.03	125.73	4.24	142.05	5.11	146.17	5.24	146.24
total	10.00	468.20	20.00	601.62	29.99	646.87	31.58	647.70

Tab6 the results of gas allocation for the oilfield with the different gas injection rates

Well group	total gas injection rate（$10^4m^3/d$）、total oil production（m^3/d）					
	20		40		60	
	q_{gi}	q_{oi}	q_{gi}	q_{oi}	q_{gi}	q_{oi}
A	16.27	803.10	23.92	954.16	31.57	1023.87
B	3.72	306.94	16.08	550.82	28.43	663.35
total	19.99	1110.04	40.00	1504.98	60.00	1687.22

CONCLUSIONS

1. Underthe shortage of the total gas injection, it is of accordance with the practicability to fields to fit only the scattered points ahead of maximum value in the gas lift performance curve.
2. Considering the global optimization of gas allocation,the well group gas-lift performance curve is defined as the relationship between the gas injection rate and the maximum oil production rate.
3. The paper consummates the well group optimization model of gas allocation and develops the oilfield optimization model of gas allocation.
4. The exterior point technique of the penalty function is presented to solve nonlinear optimization of gas allocation model,is easy to do.
5. A new approach named maximum gas allocation is proposed to guess the initial gas injection rates, allocating gas in proportion to the maximum injection rate for each well.the example shows it is feasible.
6. A method is proposed to allocate gas injection when all curves(A,B,C and D) in Fig1 exist in a group of wells.
7. A computer program is developed to perform for the optimization of gas allocation.

Reference

[1] Mayhill.T.D,1974, “ Simplified Method for Gas Lift Well Problem Identification and Diagnosis”,Paper SPE 5151,SPE 49th Annual Fall Meeting,Houston,TX.

[2] Radden,J.D.Sherman,T.A.G,and Blann,J.R,1974, “ Optimizing Gas Lift Systems”,Paper SPE 5151,SPE 49th Annual Fall Meeting,Houston,TX.

[3] Gomez.V,1974, “ Optimization of Continuous Flow Gas Lift System,”M.S.thesis,Universty of Tulsa,OK.

[4] Kanu.E,Mach,J.M,and Brown,K.E,1981, “ Economic Approach to oil Production and Gas Allocation in Continuous Gas Lift”,JPT,J.Pet.Technol,33,pp.1887-1892.

[5] N.Nishikiori,N.Redner,R.A,Doty,D.R,and Schmidt,Z,1995, “ An Improved Method for Gas Lift Allocation Optimization”,ASME J.Energy Resource,Technol,117,pp.87-92.

[6] Hong.H.T,1975, “ Effect of Variables on Optimization of Continuous Flow Gas Lift System”,M.S.thesis,University of Tulsa ,OK.

[7] Buitrago.s.etc, “ Global Optimization Techniques in Gas Allocation for Continuous Flow Gas Lift System.

[8] Kunal Dutta-Roy,SPE and James Kattapuram, “ A New Approach to Gas Lift Allocation Optimization”,SPE 38333.

[9] Gabriel A.Alarcon Carlos F.Torres,“ Global Optimization of Gas Allocation to a Group of Wells in Artificial Lift Using Nonlinear Constrained Programming”,Transaction of ASME,2002.10.VOL.124,262-268.

Appendix

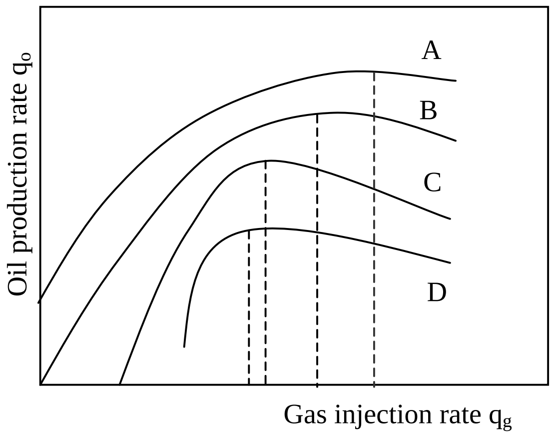

Fig.1 Type of the GLPC

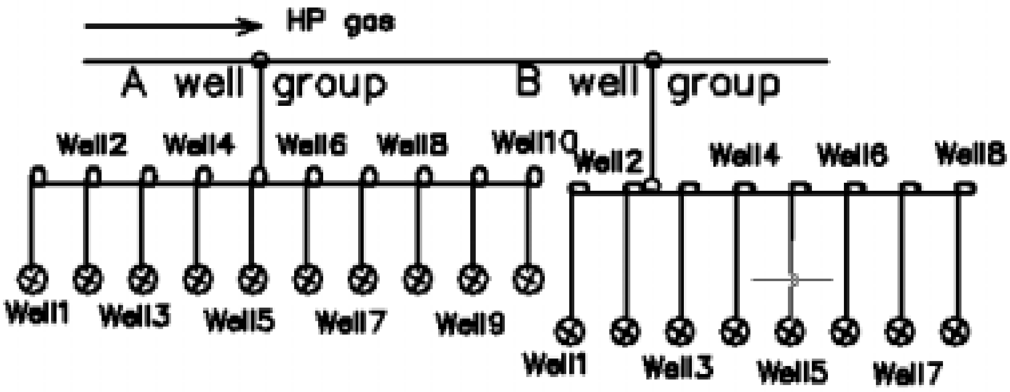

Fig.2 Network configuration for example application

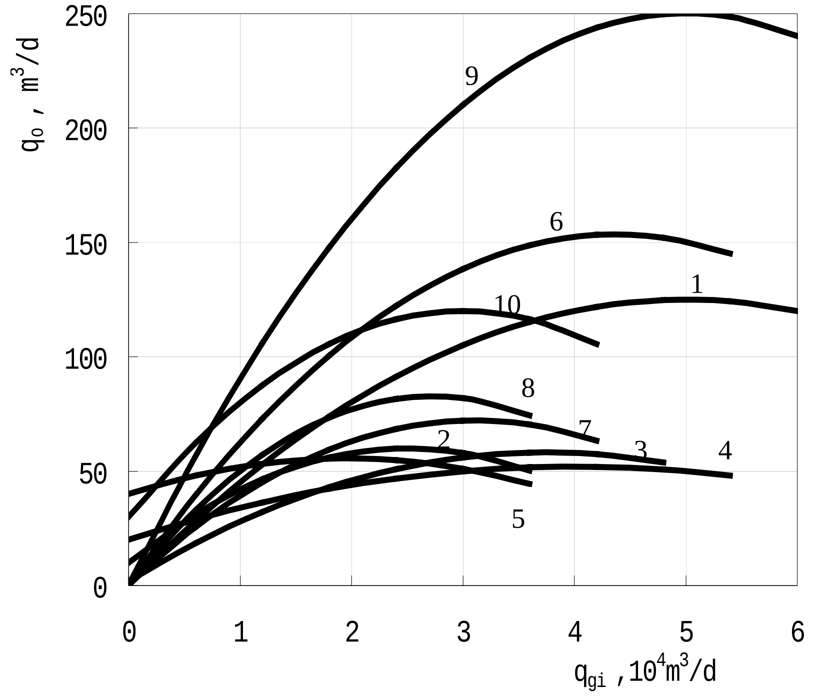

Fig.3 Production rate vs.Injection rate for a single well in well group A

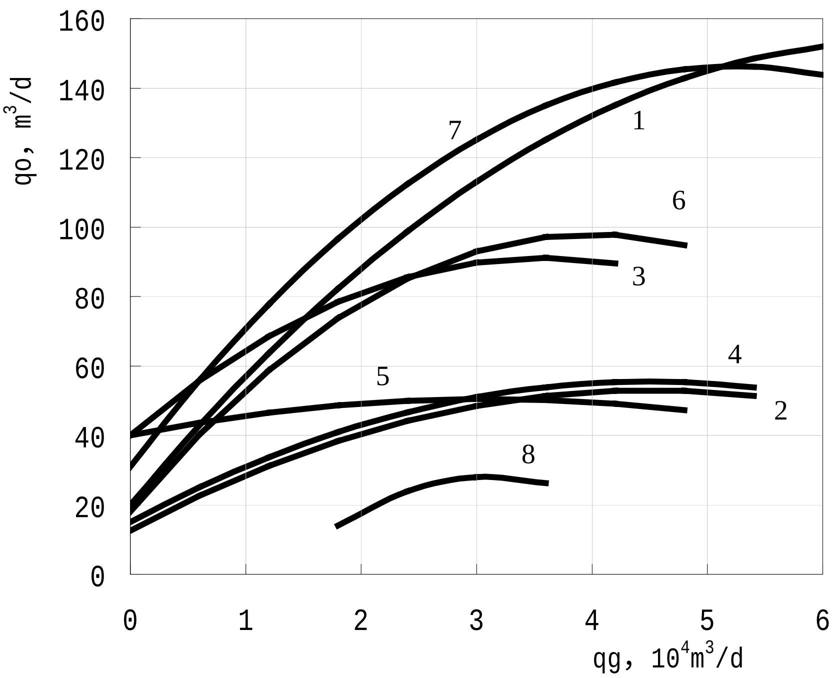

Fig.4 Production rate vs.Injection rate for a single well in well group B

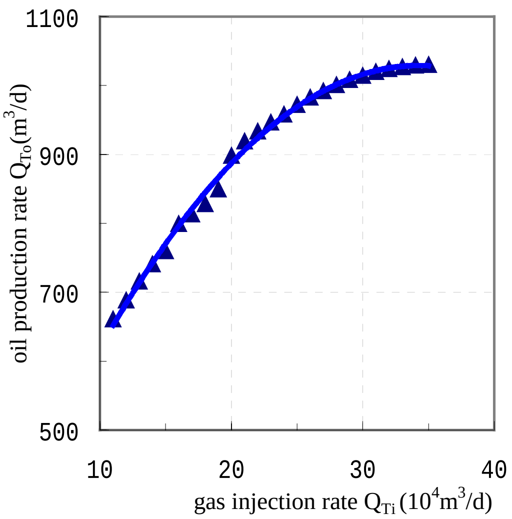

Fig.5 the well group A GLPC

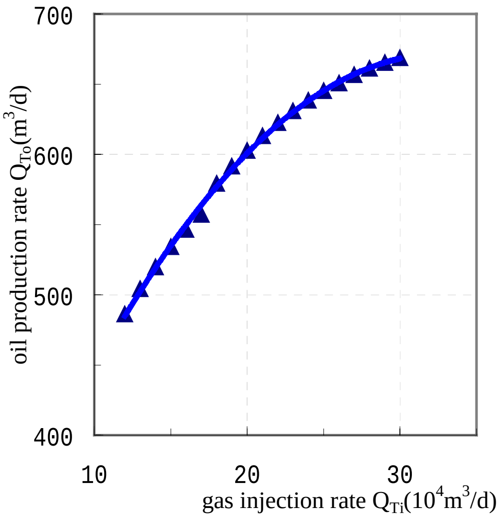

Fig.6 the well group B GLPC